Artificial Potential Fields in a complex robotic system Configuration Space

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Abstract

Artificial Potential Fields (APF) [4][5] *for robotic path planning were firstly introduced by Khatib as a solution of the basic planning problem, where a single point shaped robot with no kinematic constraints has to move from a starting point to a goal, in presence of fixed obstacles.*

In this paper, we present a mathematical tool useful for dealing with Artificial Potential Fields in configuration spaces [6] *of different kinds of complex robotic systems, such as manipulation structures and cooperating mobile robot teams, also (in both cases) in narrow environment and with agents of arbitrary shape. In general, a mathematical function* U *describing an APF will be defined on some kind of measure of scalar distance* ρ *of the actual state of the system (fully described by the Lagrangian coordinates vector* $\mathbf{q}$*) from the nearest collision configuration in the C-Space. In order to apply, in accordance to the APF paradigm, a method of minimum search in U (that can be a simple gradient descent or a more sophisticated method guided by the gradient itself), we have obviously to compute the gradient of the potential function* $\frac{\partial}{\partial \mathbf{q}} U(\rho(\mathbf{q})) = \frac{\partial U(\rho)}{\partial \rho} \frac{\partial \rho(\mathbf{q})}{\partial \mathbf{q}}$. *If the calculus of the first part of this derivative is banal, only depending on the definition of* $U(\rho)$ *, the calculus of the second part, depending from the geometry of the objects involved, is a very complex task.*

In order to solve this problem, in the following sections we thus introduce a definition of the distance measure $\rho(\mathbf{q})$ *in the C-Space and a method for its gradient evaluation in a given configuration. Then, we also present the results of the application of this method to the problem of path planning for a six joint planar manipulator in presence of obstacles, using our gradient expression as a base for a potential guided variational planning algorithm.*

1 Introduction

A path planning problem in a robotic system can have a very high level of complexity [3]. For example, we can think about sets of manipulators mounted on mobile robotic vehicles moving in two or three dimensions. They could operate on objects of any shape and dimension, in presence of moving obstacles [1]. A further complication can be given by the presence of several robotic agents that have to perform cooperative behaviours.

A popular method to deal with this complexity consists in considering a low complexity robotic path planning problem (the "basic motion planning problem") and find efficient and reliable solutions to it. In a second step, we have to extend these solutions to more and more complex environments, in an incremental way.

The *APF* method for the basic motion planning problem has its origin in the analogy with the electrostatic field theory. The model assumes that the target exerts an attractive force on the robot, and the obstacles exert a repulsive force on it.

Thus, the law of motion of the robot can be seen as a descent of the gradient of the potential field U, which is given by the sum of a repulsive field U_{rep} generated by obstacles and an attractive field U_{att} generated by the target. The motion planning is performed in iterative way: at each step, the artificial force $\mathbf{F}(\mathbf{q}) = -\nabla U(\mathbf{q})$ obtained from the potential function is taken as the most promising direction for the robot to move to.

This method is obviously affected by the problem of local minima [2], and several methods are been proposed in literature [7][8] to deal with this problem; in this case, we are generally talking about *APF guided methods*.

However, the problem of escaping from local minima exceeds the task of the present article, that is focused to provide a general method to explicit compute the generalized repulsive force $\mathbf{F}_{rep}$ in the *C-Space* of the system.

2 The gradient of an APF in a complex robotic system

2.1 The shortest distance between two agents

Consider a couple of rigid bodies A_k ($k = i, j$ where $i<j$ and $i,j=0,\dots,N$) extracted from a set of N similar objects moving in a three-dimensional space described by a fixed Cartesian coordinate system (O,X,Y,Z). On introducing the rigid body rest frame (O_k,x,y,z) the motion of each A_k is suitably described by six Lagrangian coordinates $X_k,Y_k,Z_k,\psi_k,\theta_k,\phi_k$, where X_k,Y_k,Z_k denote the components (in the fixed frame) of the vector position $\mathbf{C}_k$ of the origin O_k, namely $\mathbf{C}_k = O_k - O$, and ψ_k,θ_k,ϕ_k are the usual Euler angles.

Describe the boundary of A_k in the rest frame by its parametric representation $\mathbf{x} = \mathbf{r}_k(\xi_k,\eta_k)$ where the parameters ξ_k,η_k vary in suitable real intervals. Now, we can write the equation of the boundary of A_k in the fixed frame in the form $\mathbf{X}_k(\xi_k,\eta_k) = \mathbf{C}_k + \mathbf{r}_k(\xi_k,\eta_k)$.

Of course, due to the motion of the body A_k, the vector $\mathbf{X}_k(\xi_k,\eta_k)$ depends also on the Lagrangian coordinates that individuate the relative position of the rest frame with respect to the fixed frame. We make this dependence explicit by writing

$$\mathbf{X}_k(X_k,Y_k,Z_k,\psi_k,\theta_k,\phi_k;\xi_k,\eta_k) = \mathbf{C}_k + \mathbf{r}_k(\psi_k,\theta_k,\phi_k;\xi_k,\eta_k)$$

The components r_{kX},r_{kY},r_{kZ} di $\mathbf{r}_k$ in the fixed frame are related to the components evaluated in the rest frame through the formula

$$\begin{pmatrix} r_{kX}(\psi_k,\theta_k,\phi_k;\xi_k,\eta_k) \\ r_{kY}(\psi_k,\theta_k,\phi_k;\xi_k,\eta_k) \\ r_{kZ}(\psi_k,\theta_k,\phi_k;\xi_k,\eta_k) \end{pmatrix} = R(\psi_k,\theta_k,\phi_k) \begin{pmatrix} r_{kx}(\xi_k,\eta_k) \\ r_{ky}(\xi_k,\eta_k) \\ r_{kz}(\xi_k,\eta_k) \end{pmatrix} \tag{1}$$

where $R(\psi_k,\theta_k,\phi_k)$ denotes the rotation matrix

$$R(\psi_k,\theta_k,\phi_k) = R_z(\psi_k)R_x(\theta_k)R_z(\phi_k) = \begin{bmatrix} \cos\psi_k & -\sin\psi_k & 0 \\ \sin\psi_k & \cos\psi_k & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta_k & -\sin\theta_k \\ 0 & \sin\theta_k & \cos\theta_k \end{bmatrix} \begin{bmatrix} \cos\phi_k & -\sin\phi_k & 0 \\ \sin\phi_k & \cos\phi_k & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{2}$$

that implies

$$\mathbf{X}_k(X_k,Y_k,Z_k,\psi_k,\theta_k,\phi_k;\xi_k,\eta_k) = \mathbf{C}_k(X_k,Y_k,Z) + R(\psi_k,\theta_k,\phi_k) \begin{pmatrix} r_{kx}(\xi_k,\eta_k) \\ r_{ky}(\xi_k,\eta_k) \\ r_{kz}(\xi_k,\eta_k) \end{pmatrix} \tag{3}$$

The distance vector $\mathbf{d}_{ij}$ between an arbitrary point of the boundary of A_i and an arbitrary point of the boundary of A_j is given by the obvious relation

$$\mathbf{d}_{ij} = \mathbf{X}_i(\xi_i,\eta_i) - \mathbf{X}_j(\xi_j,\eta_j)$$

which takes the explicit form

$$\mathbf{d}_{ij}(X_i,\dots,\phi_i,X_j,\dots,\phi_j;\xi_i,\eta_i,\xi_j,\eta_j) = (\mathbf{C}_i + \mathbf{r}_i) - (\mathbf{C}_j + \mathbf{r}_j) \tag{4}$$

as shown in figure 1.

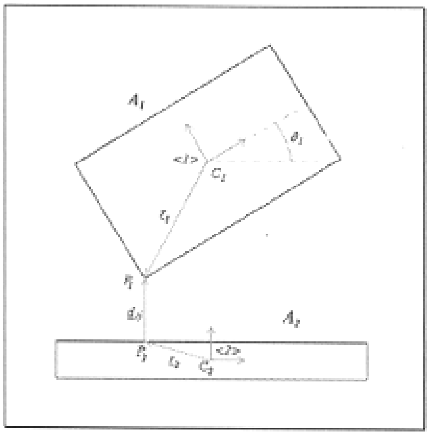

Figure 1. Shortest distance.

We now introduce the $6N$ Lagrangian coordinates component algebraic vector $\mathbf{q}$

$$\mathbf{q} = (q_1, \ldots, q_{6N})^T = \\ = (X_1, Y_1, Z_1, \psi_1, \theta_1, \phi_1, \ldots, X_N, Y_N, Z_N, \psi_N, \theta_N, \phi_N)^T \tag{5}$$

that is able to represent the entire system configuration in the *C-Space*. On assuming that the boundaries of A_i and A_j are C^1 functions (this is not restrictive, because in a real case each object boundary can be approximated by such a function with arbitrary precision), the shortest distance between the two bodies is achieved when $\mathbf{d}_{ij} \cdot \mathbf{d}_{ij}$ is minimal. Since $\mathbf{d}_{ij}$ is a function of $\xi_i, \eta_i, \xi_j, \eta_j$ this happens when

$$\frac{\partial}{\partial \xi_k}(\mathbf{d}_{ij} \cdot \mathbf{d}_{ij}) = 0, \quad \frac{\partial}{\partial \eta_k}(\mathbf{d}_{ij} \cdot \mathbf{d}_{ij}) = 0. \tag{6}$$

Of course this condition yields points of minimum, maximum and of inflection: it is implicitly understood that we refer to the points of absolute minimum. Since $\frac{\partial \mathbf{d}_{ij}}{\partial \xi_i} = \frac{\partial \mathbf{r}_i}{\partial \xi_i}$, $\frac{\partial \mathbf{d}_{ij}}{\partial \eta_i} = \frac{\partial \mathbf{r}_i}{\partial \eta_i}$ and $\frac{\partial \mathbf{d}_{ij}}{\partial \xi_j} = -\frac{\partial \mathbf{r}_j}{\partial \xi_j}$, $\frac{\partial \mathbf{d}_{ij}}{\partial \eta_j} = -\frac{\partial \mathbf{r}_j}{\partial \eta_j}$, the previous relations imply the conditions

$$\mathbf{d}_{ij} \cdot \frac{\partial \mathbf{r}_k}{\partial \xi_k} = 0, \quad \mathbf{d}_{ij} \cdot \frac{\partial \mathbf{r}_k}{\partial \eta_k} = 0 \tag{7}$$

Such a relations have a straightforward geometric interpretation: the shortest distance is achieved when the distance vector is orthogonal to both the tangent planes to the boundaries of A_k. The mathematical result is summarized by saying that the previous relation provides the values $\hat{\xi}_k, \hat{\eta}_k$ which individuate the points of shortest distance. Explicitly the shortest distance is achieved when

$$\xi_k = \hat{\xi}_k(X_i, Y_i, Z_i, \psi_i, \theta_i, \phi_i, X_j, Y_j, Z_j, \psi_j, \theta_j, \phi_j), \\ \eta_k = \hat{\eta}_k(X_i, Y_i, Z_i, \psi_i, \theta_i, \phi_i, X_j, Y_j, Z_j, \psi_j, \theta_j, \phi_j). \tag{8}$$

From now on, with a slight abuse of notation, we assume that all the relevant quantities are evaluated in correspondence of such values $\hat{\xi}_k, \hat{\eta}_k$, and we can thus define the shortest distance between A_i and A_j as

$$\rho_{ij} = \sqrt{\mathbf{d}_{ij}(\hat{\xi}_i, \hat{\eta}_i, \hat{\xi}_j, \hat{\eta}_j) \cdot \mathbf{d}_{ij}(\hat{\xi}_i, \hat{\eta}_i, \hat{\xi}_j, \hat{\eta}_j)} \tag{9}$$

2.2 The repulsive force

The overall repulsive force $\mathbf{F}_{rep}$ exerted on the configuration $\mathbf{q}$ is computed as the anti-gradient of the repulsive potential U_{rep}

$$\mathbf{F}_{rep} = \sum_{i<j} \mathbf{F}_{ij} = -\sum_{i<j} \left(\frac{\partial U_{rep}(\rho_{ij}(\mathbf{q}))}{\partial \mathbf{q}} \right)^T = \\ = -\sum_{i<j} \frac{\partial U_{rep}(\rho_{ij}(\mathbf{q}))}{\partial \rho_{ij}} \left(\frac{\partial \rho_{ij}}{\partial \mathbf{q}} \right)^T = -\sum_{i<j} U'_{rep}(\rho_{ij}) \left(\frac{\partial \rho_{ij}}{\partial \mathbf{q}} \right)^T \tag{10}$$

To compute the repulsive force $\mathbf{F}_{ij}$ that acts on two generic objects A_i and A_j, we have to calculate the derivative of ρ_{ij} with respect to $\mathbf{q}$

$$\frac{\partial \rho_{ij}}{\partial \mathbf{q}} = \frac{\partial}{\partial \mathbf{q}} \sqrt{\mathbf{d}_{ij} \cdot \mathbf{d}_{ij}} = \frac{\mathbf{d}_{ij}}{\rho_{ij}} \frac{\partial \mathbf{d}_{ij}}{\partial \mathbf{q}} \tag{11}$$

The repulsive force $\mathbf{F}_{ij}$ between A_i and A_j is so a formal *6N* component column vector whose components $F_{ij}^{\alpha}, \alpha = 1,..,6N$ are defined as

$$F_{ij}^{\alpha} = -U'_{rep}(\rho_{ij}) \frac{\mathbf{d}_{ij}}{\rho_{ij}} \frac{\partial \mathbf{d}_{ij}}{\partial q^{\alpha}} = \\ = -U'_{rep}(\rho_{ij}) \frac{\mathbf{d}_{ij}}{\rho_{ij}} \frac{\partial [(\mathbf{C}_i + \mathbf{r}_i) - (\mathbf{C}_j + \mathbf{r}_j)]}{\partial q^{\alpha}} \tag{12}$$

The present task is that of evaluating $\mathbf{F}_{ij}$ explicitly. We calculate first the derivatives

$$\mathbf{d}_{ij} \cdot \frac{\partial(\mathbf{C}_i + \mathbf{r}_i)}{\partial q_{\alpha}}, \quad \alpha = 6j-5, \ldots, 6j \\ \mathbf{d}_{ij} \cdot \frac{\partial(\mathbf{C}_j + \mathbf{r}_j)}{\partial q_{\alpha}}, \quad \alpha = 6i-5, \ldots, 6i \tag{13}$$

observing that

$$\frac{\partial \mathbf{d}_{ij}}{\partial q_{\alpha}} = 0 \ \text{per} \ \alpha \notin \{6i-5, \ldots, 6i, 6j-5, \ldots, 6j\} \tag{14}$$

Due to the obvious relations

$$\frac{\partial \mathbf{C}_i}{\partial q_{\alpha}} = 0, \quad \alpha = 6j-5, \ldots, 6j, \\ \frac{\partial \mathbf{C}_j}{\partial q_{\alpha}} = 0, \quad \alpha = 6i-5, \ldots, 6i \tag{15}$$

and using (7) we get

$$\mathbf{d}_{ij} \cdot \frac{\partial(\mathbf{C}_i + \mathbf{r}_i)}{\partial q_{\alpha}} = \mathbf{d}_{ij} \cdot \frac{\partial \mathbf{r}_i}{\partial \xi_i} \frac{\partial \xi_i}{\partial q_{\alpha}} + \mathbf{d}_{ij} \cdot \frac{\partial \mathbf{r}_i}{\partial \eta_i} \frac{\partial \eta_i}{\partial q_{\alpha}} = 0 \\ \alpha = 6j-5, \ldots, 6j \tag{16}$$

$$\mathbf{d}_{ij} \cdot \frac{\partial(\mathbf{C}_j + \mathbf{r}_j)}{\partial q_{\alpha}} = \mathbf{d}_{ij} \cdot \frac{\partial \mathbf{r}_j}{\partial \xi_j} \frac{\partial \xi_j}{\partial q_{\alpha}} + \mathbf{d}_{ij} \cdot \frac{\partial \mathbf{r}_j}{\partial \eta_j} \frac{\partial \eta_j}{\partial q_{\alpha}} = 0, \\ \alpha = 6i-5, \ldots, 6i \tag{17}$$

Thus, the quantities F_{ij}^{α} reduce to

$$F_{ij}^{\alpha} = -U'_{rep}(\rho_{ij})\frac{\mathbf{d}_{ij}}{\rho_{ij}} \cdot \frac{\partial}{\partial q_{\alpha}}(\mathbf{C}_i + \mathbf{r}_i), \quad \alpha = 6i-5,\ldots,6i \qquad (18)$$

$$F_{ij}^{\alpha} = U'_{rep}(\rho_{ij})\frac{\mathbf{d}_{ij}}{\rho_{ij}} \cdot \frac{\partial}{\partial q_{\alpha}}(\mathbf{C}_j + \mathbf{r}_j), \quad \alpha = 6j-5,\ldots,6j$$

A straightforward calculation yields

$$\begin{aligned} F_{ij}^{6i-5} &= -U'_{rep}(\rho_{ij})\frac{\mathbf{d}_{ij}}{\rho_{ij}} \cdot \frac{\partial \mathbf{C}_i}{\partial q_{6i-5}} = \\ &= -U'_{rep}(\rho_{ij})\left(\frac{\mathbf{d}_{ij}}{\rho_{ij}} \cdot \frac{\partial \mathbf{r}_i}{\partial \xi_i}\frac{\partial \xi_i}{\partial q_{6i-5}} + \frac{\mathbf{d}_{ij}}{\rho_{ij}} \cdot \frac{\partial \mathbf{r}_i}{\partial \eta_i}\frac{\partial \eta_i}{\partial q_{6i-5}}\right) = \\ &= -U'_{rep}(\rho_{ij})\frac{d_{ijX}}{\rho_{ij}} \end{aligned} \qquad (19)$$

having used again relations (7). Analogously

$$\begin{aligned} &F_{ij}^{6i-4} = -U'_{rep}(\rho_{ij})\frac{d_{ijY}}{\rho_{ij}}, \quad F_{ij}^{6i-3} = -U'_{rep}(\rho_{ij})\frac{d_{ijZ}}{\rho_{ij}}, \\ &F_{ij}^{6j-5} = U'_{rep}(\rho_{ij})\frac{d_{ijX}}{\rho_{ij}}, \quad F_{ij}^{6j-4} = U'_{rep}(\rho_{ij})\frac{d_{ijY}}{\rho_{ij}}, \\ &F_{ij}^{6j-3} = U'_{rep}(\rho_{ij})\frac{d_{ijZ}}{\rho_{ij}}. \end{aligned} \qquad (20)$$

Consider now the quantity F_{ij}^{6i-2} and remember that $q_{6i-2} = \psi_i$. Since $\frac{\partial \mathbf{C}_i}{\partial q_{6i-2}} = 0$, in view of (1) and employing matrix notation, we obtain

$$\begin{aligned} F_{ij}^{6i-2} &= -\frac{U'_{rep}(\rho_{ij})}{\rho_{ij}}\mathbf{d}_{ij} \cdot \frac{\partial \mathbf{r}_i}{\partial \psi_i} = \\ &= -\frac{U'_{rep}(\rho_{ij})}{\rho_{ij}}(d_{ijX}\, d_{ijY}\, d_{ijZ})\frac{\partial R(\psi_i)}{\partial \psi_i}R(\theta_i)R(\phi_i)\begin{pmatrix} r_{ix}(\xi_i,\eta_i) \\ r_{iy}(\xi_i,\eta_i) \\ r_{iz}(\xi_i,\eta_i) \end{pmatrix} \end{aligned} \qquad (21)$$

having used once more relation (7). On inverting relation (1) we have

$$\begin{pmatrix} r_{ix} \\ r_{iy} \\ r_{iz} \end{pmatrix} = R^T(\psi_i,\theta_i,\phi_i)\begin{pmatrix} r_{iX} \\ r_{iY} \\ r_{iZ} \end{pmatrix} \qquad (22)$$

Hence, formula (19) becomes

$$\begin{aligned} F_{ij}^{6i-2} &= -\frac{U'_{rep}(\rho_{ij})}{\rho_{ij}}(d_{ijX}\, d_{ijY}\, d_{ijZ})\frac{\partial R(\psi_i)}{\partial \psi_i} \cdot \\ &\cdot R(\theta_i)R(\phi_i)R^T(\phi_i)R^T(\theta_i)R^T(\phi_i)\begin{pmatrix} r_{iX} \\ r_{iY} \\ r_{iZ} \end{pmatrix} \end{aligned} \qquad (23)$$

We have

$$\frac{\partial R(\psi_i)}{\partial \psi_i}R^T(\psi_i) = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{pmatrix} \qquad (24)$$

so we conclude with the formula

$$F_{ij}^{6i-2} = -\frac{U'_{rep}(\rho_{ij})}{\rho_{ij}}\mathbf{r}_i \times \mathbf{d}_{ij} \cdot \mathbf{e}_3 \qquad (25)$$

where $\mathbf{e}_3$ denotes the unit vector of the Z axis. Explicit calculation gives

$$F_{ij}^{6i-1} = -\frac{U'_{rep}(\rho_{ij})}{\rho_{ij}}\mathbf{r}_i \times \mathbf{d}_{ij} \cdot \mathbf{N}_i \qquad (26)$$

where, in the usual formalism of Euler angles, $\mathbf{N}_k$ and $\hat{\mathbf{e}}_3^k$ denote the unit vector of the node axis and of the z axis of the rest frame associated to the body A_k.

To sum up all the results obtained so far we can write $\mathbf{F}_{ij}$ as the column vector

$$\begin{aligned} \mathbf{F}_{ij} = -\frac{U'_{rep}(\rho_{ij})}{\rho_{ij}}\Big[&\underset{1}{0} \;\ldots\; \underset{6(i-1)}{0} \;\; \underset{6i-5}{d_{ijX}} \;\; \underset{6i-4}{d_{ijY}} \;\; \underset{6i-3}{d_{ijZ}} \;\; \underset{6i-2}{\mathbf{r}_i \times \mathbf{d}_{ij} \cdot \mathbf{e}_3} \\ &\underset{6i-1}{\mathbf{r}_i \times \mathbf{d}_{ij} \cdot \mathbf{N}_i} \;\; \underset{6i}{\mathbf{r}_i \times \mathbf{d}_{ij} \cdot \hat{\mathbf{e}}_3^i} \;\; \underset{6i+1}{0} \;\ldots\; \underset{6(j-1)}{0} \;\; \underset{6j-5}{-d_{ijX}} \;\; \underset{6j-4}{-d_{ijY}} \;\; \underset{6j-3}{-d_{ijZ}} \\ &\underset{6j-2}{-\mathbf{r}_j \times \mathbf{d}_{ij} \cdot \mathbf{e}_3} \;\; \underset{6j-1}{-\mathbf{r}_j \times \mathbf{d}_{ij} \cdot \mathbf{N}_j} \;\; \underset{6j}{-\mathbf{r}_j \times \mathbf{d}_{ij} \cdot \hat{\mathbf{e}}_3^j} \;\; \underset{6j+1}{0} \;\ldots\; \underset{6N}{0} \Big]^T \end{aligned} \qquad (27)$$

The six components of the generalized force $\mathbf{F}_{ij}$ in places i act on A_i configuration, whilst those in places j act on A_j configuration.

Finally, we can express the resultant force $\mathbf{F}_{rep}$ as

$$\mathbf{F}_{rep} = \begin{bmatrix} \mathbf{f}_1 \\ \vdots \\ \mathbf{f}_N \end{bmatrix} = \sum_{\substack{i,j=0..N \\ i<j}} \mathbf{F}_{ij} \qquad (28)$$

where $\mathbf{f}_k$ (k=1,…,N) is the generalized six components force that acts on A_k configuration. In particular, may be interesting to focus that the fourth, fifth and sixth components in each of the column vectors are the components of the total torque acting on A_k, thus introducing a rotational effect that changes agent attitude $(\psi_k, \theta_k, \phi_k)$ according to the repulsive field, with the purpose of keeping the agent far from contact configurations in the *C-Space*.

As we can see, also if our reasoning is based on the configuration space, the calculus of the generalized repulsive force can be decoupled in a distributed algorithm, where each agent only requires the knowledge about the minimum distance vectors between itself and other agents and obstacles.

All information required by the algorithm can be deducted, at each time step, directly from the workspace by simple geometric considerations, without explicitly performing the task (unmanageable in the majority of real cases) of mapping the obstacles in the *C-Space*.

2.3 The distance in the *C-Space*

A *configuration obstacle CO* is defined as follows

$$CO_{ij} = \{\mathrm{p} \in \text{C-Space}: A_i(\mathrm{p}) \cap A_j(\mathrm{p}) \neq 0\} \quad i, j \in 0..N, i < j$$
$$CO = \bigcup_{i,j \in 0..N, i<j} CO_{ij} \tag{29}$$

where A_0 is the union set of all the fixed obstacles in the *Work Space*, and $A_1, \ldots, A_N$ are the N agents. Notice that we consider collision configurations not only particular configurations in which we have contact between an agent and a fixed obstacle, but also configurations in which a collision happens between two agents. The impossibility of mapping obstacles directly in the *C-Space*, derives directly from the necessity of taking into account all the configurations in which an agent becomes an obstacle for other agents.

CO is the union of all the sets of configurations CO_{ij} in which only two objects collide (i.e. a couple of agents or a couple agent-obstacle). In fact, the configurations in which there is a contemporary collision among three or more objects are implicitly considered, since they are a subset of the configurations in which only two agents collide.

We could define the distance ρ of the system from *CO* as the distance between the configuration q and the closest collision configuration belonging to *CO* in the *C-Space*. Following this definition, we should assume $\rho = \min_{ij}(\rho_{ij})$ and consequently compute the repulsive potential U_{rep} and the repulsive force $\mathbf{F}_{rep}$. In other words, we are focusing our attention on the couple of objects (denoted by i and j) having the most critical configuration in the sense of the navigation problem.

Notice that, although the calculus of the repulsive force exerted on q by the closest collision configuration is, in most cases, unmanageable in the *C-Space*, it results very simple in the work space by simple geometric considerations, because we have only to consider the couple of objects in the most critical configuration to avoid any collision.

However, to obtain a safer and smoother behaviour, we can express ρ in a different way, by considering the most critical configuration for each set CO_{ij}. That is, for each agent, we take into account the minimum distance from any other object within a given distance.

In the following section, we will see that the definition that we choose for the potential function, that is responsible of all the forces that act on the system, implicitly defines the concept of distance in the *C-Space*.

2.4 An example repulsive potential function

An example of repulsive artificial potential function with "good properties" that acts between two objects A_i and A_j is the following:

$$U_{rep}(\rho_{ij}) = \begin{cases} \frac{1}{2}\eta\left(\frac{1}{\rho_{ij}(\mathbf{q})} - \frac{1}{\rho_0}\right)^2 & \rho_{ij} < \rho_0 \\ 0 & \rho_{ij} \geq \rho_0 \end{cases} \tag{30}$$

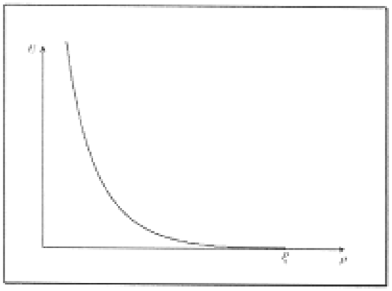

Figure 2. The potential function.

where ρ_{ij} is the shortest distance between A_i and A_j, and ρ_0 is the maximum influence distance, beyond which there is no interaction between objects.

Finally, we define ρ by having appeal to the superimposition principle for the potential field U_{rep} and, consequently, for the overall repulsive force $\mathbf{F}_{rep}$:

$$U_{rep}(\rho) = \sum_{ij} U_{rep}(\rho_{ij}) \quad i, j \in 0..N, i < j \tag{31}$$

According to the definition of the potential function given in this section, we obtain, with $\rho < \rho_0$

$$\frac{1}{2}\left(\frac{1}{\rho}-\frac{1}{\rho_0}\right)^2=\sum_{ij}\frac{1}{2}\left(\frac{1}{\rho_{ij}}-\frac{1}{\rho_0}\right)^2$$
$$\Rightarrow \rho \overset{def}{=} \frac{1}{\dfrac{1}{\rho_0}+\sqrt{\sum_{ij}\left(\dfrac{1}{\rho_{ij}}-\dfrac{1}{\rho_0}\right)^2}} \qquad i,j\in 0..N,\; i<j \tag{32}$$

As a final remark, we have shown that the choice of a particular repulsive potential function implicitly defines a distance function $\rho(\mathbf{q})$ in the configuration space of the system, i.e. a measure of the distance of the actual system configuration from a collision configuration between agents or between an agent and an obstacle in the *C-Space*.

It is possible to demonstrate that in this case $\rho \le \min_{ij}(\rho_{ij})$, thus having a behaviour safer than the choice of $\rho = \min_{ij}(\rho_{ij})$. Moreover, as a consequence of the superimposition principle, there will be components in $\mathbf{F}_{rep}$ for all the couples of objects in mutual dangerous configuration.

3 An example application to a robotic arm

In the sequence in figure 3, we present the results obtained by simulating a 6 dof planar manipulator that has to move from a starting to a goal configuration, avoiding obstacles on its path.

The mathematical tools described in the previous sections are used as a base for a potential guided variational planning method that minimize a cost function, defined on the global path of the manipulator.

A local method to escape from local minima, inspired to the *Micronavigation* [7] algorithm, is added to integrate the pure gradient descent method applied in order to minimize the global cost function.

Each link of the robotic arm is treated as a separate agent, with its own shape and kinematic constraints. Non-holonomic constraints are taken into account by introducing in the cost function some repulsive potential terms, that increases more and more while the system configuration get closer to configuration areas prohibited by the constraints.

Figure 3. Robotic arm path planning.

We can observe that the global cost function minimization method, applied to the problem of the path planning for the robotic arm, gives to the system a "look forward" behaviour, that force the arm to fold on itself before getting close to the obstacle.

4 Conclusion

In this paper, we presented a mathematical tool that can be used as a base for a wide class of potential guided path planning methods, in a large number of robotic structures, as manipulators and more general multi agent systems.

These methods allow to deal with the intrinsic complexity of the path planning problem for multi agents (of any shape and in narrow environments) by decoupling the interactions in distributed algorithms, and avoid the complexity of interaction mapping in the *C-Space* by deducting them directly from the work space of the robotic system.

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