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A LAGRANGIAN DENSITY FOR THE DYNAMICS OF ELASTIC DIELECTRICS

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Abstract—The equations governing the behaviour of an elastic dielectric in the quasi-static approximation are analysed in connection with the inverse problem of the calculus of variations. First, through compatibility with the potential conditions, the most general constitutive equations are derived which allow the model to admit a variational formulation. Then the explicit form of the Lagrangian density is determined. Consistent with the adoption of Lagrangian coordinates, the Lagrangian density ascribes to the dielectric the structure of a particle-like system.

1. INTRODUCTION

There are several aspects which motivate the wide literature of elastic dielectrics (see, e.g., [1] and references therein). Within the electrodynamics of deformable media, the elastic dielectric constitutes a very simple model in which magnetization is disregarded; that is why many objections against some methods employed for the description of the electromagnetic interaction do not apply to the elastic dielectric scheme. On the other hand the simplicity of the model does not prevent it from describing various interesting phenomena based on piezo-electric, photo-elastic, and electro-optical properties [2]. Mathematically, such phenomena are modelled by non-linear constitutive functions, which is consistent with the general feature that, although Maxwell's equations for the electromagnetic field in vacuum are linear, in material media they are not. This intrinsic non-linearity of the elastic dielectric model indicates that it is more appropriate to have recourse to methods of non-linear analysis instead of adopting linearised approximations. In our opinion new analytical methods and computation procedures are likely to be gained if the equations modelling the elastic dielectric are shown to arise from a genuine variational formulation in the sense that they are the Euler–Lagrange equations of a proper functional.

To ascertain whether a given set of differential equations admits a variational formulation is a topic pertaining to the inverse problem of the calculus of variations. As is well known, a general formal answer to the inverse problem is delivered by a theorem of Vainberg [3]; a paper of ours [4] exhibits detailed conditions (potentialness conditions) ensuring the existence of a variational principle. Such potentialness conditions are sufficient for answering completely the inverse problem if, as we do here, we are interested in the differential equations but not in the boundary conditions (formal differential operators; see, e.g., [5]).

In this paper we prove that the equations for the elastic dielectric in the quasi-static approximation, described through Lagrangian coordinates, admit a genuine variational formulation. Our procedure is as follows. As a starting point we let the stress tensor and the electric displacement be functions of the deformation gradient and the electric field. Next these response functions are required to satisfy the potentialness conditions; so we arrive at the most general form of the response functions compatible with the existence of a variational formulation. The final step is merely the evaluation of the Lagrangian density. Consistently with the adoption of Lagrangian coordinates, the Lagrangian density turns out to have the structure typical of particle-like systems.

2. BALANCE EQUATIONS IN THE QUASI-STATIC APPROXIMATIONS

The complete set of equations describing the electrodynamics of deformable bodies of finite extent has been investigated extensively (see, e.g., [1, 6–8] and references therein). Notwithstanding this, it seems that there is no universal agreement about the detailed form of the terms entering the momentum balance and the energy balance. On the other hand, a variational formulation for the electrodynamics of deformable bodies appears to be a

formidable task. This suggests that we start with a simpler framework like that of elastic dielectrics within the quasi-static approximation where the magnetization is neglected as well as the time rate of change of magnetic induction and of electric displacement.

With the purpose of describing the finite motion of the elastic dielectric we label each particle of the body by its position $\mathbf{X}$ in a suitable reference configuration; so $\mathbf{x}(\mathbf{X}, t)$ is the position of the particle $\mathbf{X}$ at time t, $\dot{\mathbf{x}} = \partial \mathbf{x}(\mathbf{X}, t)/\partial t$ is the velocity, $\mathbf{F} = \operatorname{Grad} \mathbf{x}(\mathbf{X}, t)$ is the deformation gradient, the operator $\operatorname{Grad} = \partial/\partial \mathbf{X}$ denoting the gradient with respect to material coordinates. Owing to the conservation of mass, the mass density ρ is determined through the motion by

$$\rho J = \rho_0(\mathbf{X})$$

where $J = \det \mathbf{F}$ and ρ_0 is the mass density in the reference configuration. In connection with a Lagrangian description it is convenient to consider the reference measures $\mathscr{E}$ and $\mathscr{D}$ of the electric field $\mathbf{E}$ and the electric displacement $\mathbf{D}$ [1, 9], namely

$$\mathscr{E} = \mathbf{FE}, \qquad \mathscr{D} = J\mathbf{F}^{-1}\mathbf{D},$$

the direct tensor notation being used as usual; the magnetic quantities $\mathscr{H}, \mathscr{B}$ are irrelevant in the quasi-static approximation. Analogously, the stress of the body is conveniently described through the first Piola–Kirchhoff stress tensor $\mathbf{S}$. These fields enter the pertinent balance equations in the form

$$\begin{aligned} \rho_0 \ddot{\mathbf{x}} &= \operatorname{Div} \mathbf{S} + \rho_0 \mathbf{b}, \\ \operatorname{Div} \mathscr{D} &= \mu_0, \\ \operatorname{Curl} \mathscr{E} &= \mathbf{0}, \end{aligned} \tag{2.1}$$

where $\mathbf{b}$ is the body force and μ_0 is the (reference) charge density. Owing to the condition $(2.1)_3$ the electric field $\mathscr{E}$ is potential; hence there exists a scalar function ϕ which allows $(2.1)_3$ to be replaced with

$$\mathscr{E} = -\operatorname{Grad} \phi. \tag{2.2}$$

It is worth remarking that the system of equations (2.1) does not involve the balance of energy whose form, though, is not agreed upon [1]. Physically, disregarding the balance of energy means that the temperature field is not considered. Besides being motivated by simplicity reasons, this neglect is related to conceptual difficulties in finding a variational principle from which the entire system of equations may be derived.

We have now to characterise the nature of the body under consideration through the introduction of a suitable set of constitutive equations. As our attention is confined to elastic dielectrics, we assume that the response functions $\mathbf{S}$ and $\mathscr{D}$ depend on the deformation gradient $\mathbf{F}$, the electric field $\mathscr{E}$, and, possibly, on the independent variables $\mathbf{X}, t$; meanwhile we regard $\mathbf{b}$, ρ_0, and μ_0 as assigned functions of $\mathbf{X}$ and t. Then, in view of (2.2), we are led to account for the behaviour of the dielectric via the system of equations

$$\begin{aligned} \rho_0 \ddot{\mathbf{x}} - \operatorname{Div} \mathbf{S}(\operatorname{Grad} \mathbf{x}, \operatorname{Grad} \phi, \mathbf{X}, t) - \rho_0 \mathbf{b}(\mathbf{X}, t) &= \mathbf{0}, \\ \operatorname{Div} \mathbf{D}(\operatorname{Grad} \mathbf{x}, \operatorname{Grad} \phi, \mathbf{X}, t) - \mu_0(\mathbf{X}, t) &= 0, \end{aligned} \tag{2.3}$$

in the unknown functions $\mathbf{x}(\mathbf{X}, t)$, $\phi(\mathbf{X}, t)$.

3. CONDITIONS FOR S AND $\mathscr{D}$ TO ALLOW A VARIATIONAL FORMULATION

On the basis of the general theory developed in [4] we assert that a second order system of the form

$$f_\Gamma(u_\Omega, u_{\Omega,\alpha}, u_{\Omega,\alpha\beta}, y_\alpha) = 0, \Gamma, \Omega = 1, \ldots, m, \alpha, \beta = 1, \ldots, n, \tag{3.1}$$

in the unknown functions $u_\Omega = u_\Omega(y_\alpha)$, admits a variational formulation if and only if

$$\frac{\partial f_\Gamma}{\partial u_{\Omega,\alpha\beta}} = \frac{\partial f_\Omega}{\partial u_{\Gamma,\alpha\beta}}, \tag{3.2}$$

$$\frac{\partial f_\Gamma}{\partial u_{\Omega,\alpha}} - \left(\frac{\partial f_\Gamma}{\partial u_{\Omega,\alpha\beta}}\right),_\beta = -\left[\frac{\partial f_\Omega}{\partial u_{\Gamma,\alpha}} - \left(\frac{\partial f_\Omega}{\partial u_{\Gamma,\alpha\beta}}\right),_\beta\right], \tag{3.3}$$

$$\frac{\partial f_\Gamma}{\partial u_\Omega} - \frac{1}{2}\left[\frac{\partial f_\Gamma}{\partial u_{\Omega,\alpha}} - \left(\frac{\partial f_\Gamma}{\partial u_{\Omega,\alpha\beta}}\right),_\beta\right],_\alpha = \frac{\partial f_\Omega}{\partial u_\Gamma} - \frac{1}{2}\left[\frac{\partial f_\Omega}{\partial u_{\Gamma,\alpha}} - \left(\frac{\partial f_\Omega}{\partial u_{\Gamma,\alpha\beta}}\right),_\beta\right],_\alpha; \tag{3.4}$$

the summation convention is in force and a comma followed by a Greek letter, α say, denotes differentiation with respect to y_α, $\alpha = 1,\ldots,n$. In the particular case when the functions f_Γ depend on the derivatives $u_{\Omega,\alpha}$, $u_{\Omega,\alpha\beta}$ but not on the values u_Ω, namely $\partial f_\Gamma/\partial u_\Omega = 0$, then an immediate comparison between (3.3) and (3.4) yields

$$\left[\frac{\partial f_\Gamma}{\partial u_{\Omega,\alpha}} - \left(\frac{\partial f_\Gamma}{\partial u_{\Omega,\alpha\beta}}\right),_\beta\right],_\alpha = 0. \tag{3.5}$$

Return now to the system (2.3) and observe that it may be written as

$$\begin{aligned} \chi_i &:= \rho_0 \ddot{x}_i - \frac{\partial S_{i(M}}{\partial x_{p,Q)}} x_{p,QM} - \frac{\partial S_{i(M}}{\partial \phi_{,Q)}} \phi_{,QM} - \frac{\partial S_{iM}}{\partial X_M} - \rho_0 b_i = 0, \\ \Phi &:= \frac{\partial \mathscr{D}_{(M}}{\partial x_{p,Q)}} x_{p,QM} + \frac{\partial \mathscr{D}_{(M}}{\partial \phi_{,Q)}} \phi_{,QM} + \frac{\partial \mathscr{D}_M}{\partial X_M} - \mu_0 = 0; \end{aligned} \tag{3.6}$$

we use Cartesian tensor notation, the convention that capital indices refer to coordinates and lower case indices to present coordinates of material points, and the convention that round (square) brackets, enclosing a pair of indices, denote symmetrization (skew-symmetrization). Let $u_i = x_i\,(i = 1,2,3), u_4 = \phi, f_i = \chi_i\,(i = 1,2,3), f_4 = \Phi$, and $y_M = X_M\,(M = 1,2,3), y_4 = t$; the second order system (3.6) admits a variational formulation if and only if there exist constitutive functions $\mathbf{S}, \mathscr{D}$ making the conditions (3.2)–(3.4) identically satisfied. In fact, because χ_i and Φ are independent of x_i and ϕ, the condition (3.4) reduces to the simpler condition (3.5).

Before commencing the detailed analysis of (3.2)–(3.4) in connection with the system (3.6) we point out that the identity

$$\frac{\partial}{\partial u_{\Gamma,\alpha}} h_{M,M} = \left(\frac{\partial h_M}{\partial u_{\Gamma,\alpha}}\right),_M \tag{3.7}$$

holds for any vector function $\mathbf{h}$ which depends on $u_{\Gamma,\alpha}$ and $\mathbf{X}, t$ (and, possibly also on higher order derivatives).

The analysis begins with the case $\Gamma, \Omega = 1,2,3$. Observe first that

$$\frac{\partial \chi_i}{\partial x_{p,QM}} = -\frac{\partial S_{i(M}}{\partial x_{p,Q)}},$$

and that, in view of the identity (3.7) for $h_M = S_{1M}, S_{2M}, S_{3M}$,

$$\frac{\partial \chi_i}{\partial x_{p,Q}} = -\frac{\partial}{\partial x_{p,Q}} S_{iM,M} = -\left(\frac{\partial S_{iM}}{\partial x_{p,Q}}\right),_M.$$

Therefore we obtain

$$\frac{\partial \chi_i}{\partial x_{p,Q}} - \left(\frac{\partial \chi_i}{\partial x_{p,QM}}\right),_M = \left(\frac{\partial S_{i[Q}}{\partial x_{p,M]}}\right),_M.$$

Then, as a consequence of (3.2) and (3.3), we find that

$$\frac{\partial S_{i[M}}{\partial x_{p,Q)}} - \frac{\partial S_{p[M}}{\partial x_{i,Q)}} = 0, \tag{3.8}$$

$$\left(\frac{\partial S_{i[M}}{\partial x_{p,Q]}} + \frac{\partial S_{p[M}}{\partial x_{i,Q]}}\right)_{,M} = 0. \tag{3.9}$$

It is a simple matter to see that the integration of (3.9) yields

$$\frac{\partial S_{i[M}}{\partial x_{p,Q]}} + \frac{\partial S_{p[M}}{\partial x_{i,Q]}} = 2\varepsilon_{MQR}\psi_{ip,R} \tag{3.10}$$

where $\psi_{ip} = \psi_{(ip)}$ is an arbitrary tensor function of $\mathbf{X}, t$ and ε_{MQR} is the Levi–Civita symbol. Addition of (3.8) and (3.10) gives

$$\frac{\partial S_{iM}}{\partial x_{p,Q}} - \frac{\partial S_{pQ}}{\partial x_{i,M}} = 2\varepsilon_{MQR}\psi_{ip,R}.$$

This implies the existence of a scalar function $\Sigma = \Sigma(\text{Grad}\,\mathbf{x}, \text{Grad}\,\phi, \mathbf{X}, t)$ such that

$$S_{iM} = \frac{\partial \Sigma}{\partial x_{i,M}} + \varepsilon_{MQR}\psi_{ip,R}x_{p,Q}. \tag{3.11}$$

It remains now to ascertain whether or not the conditions (3.5), (3.8), (3.9)—and (3.10)—place further restrictions on Σ and ψ_{ip}. In fact, direct substitution shows that the function (3.11) satisfies identically such conditions.

It is worth remarking that the above analysis does not involve the term $\rho_0\ddot{x}_i$, appearing in χ_i, because it gives rise to identities only.

Let now $\Gamma = \Omega = 4$. A procedure similar to the previous one allows us to find that the compatibility of Φ with (3.2), (3.3) and (3.5) implies that the function $\mathscr{D}$ is expressed as

$$\mathscr{D}_M = \frac{\partial \Lambda}{\partial \phi_{,M}} + \varepsilon_{MQR}\Xi_{,R}\phi_{,Q} \tag{3.12}$$

where $\Lambda = \Lambda(\text{Grad}\,\mathbf{x}, \text{Grad}\,\phi, \mathbf{X}, t)$ and $\Xi = \Xi(\mathbf{X}, t)$ are arbitrary scalar functions.

As the last step we have to examine the case $\Gamma = 1, 2, 3$, $\Omega = 4$ (and vice versa). Since we know the expressions (3.11), (3.12) for the functions $\mathbf{S}, \mathscr{D}$, our task consists in determining further possible restrictions on $\Sigma, \psi_{ip}, \Lambda, \Xi$. A straightforward exploitation of (3.2), (3.3) and (3.5) leads to the relationship

$$\Sigma + \Lambda = \sigma + \lambda - \varepsilon_{MQR}\Theta_{i,R}x_{i,M}\phi_{,Q} \tag{3.13}$$

where $\sigma = \sigma(\text{Grad}\,\phi, \mathbf{X}, t)$, $\lambda = \lambda(\text{Grad}\,\mathbf{x}, \mathbf{X}, t)$, $\Theta_i = \Theta_i(\mathbf{X}, t)$ are arbitrary functions on their arguments.

In connection with σ and λ it is of interest to point out that the transformation $\Sigma \to \Sigma + \sigma$, $\Lambda \to \Lambda + \lambda$, leaving $\mathbf{S}$ and $\mathscr{D}$ unaffected, may be viewed as a gauge transformation. So, setting $\sigma = 0$, $\lambda = 0$ amounts to choosing a particular gauge.

4. THE LAGRANGIAN DENSITY

Since the functions (3.11) and (3.12), together with the restriction (3.13), make equations (3.6) meet the potentialness conditions (3.2)–(3.4) we are now in a position to determine the corresponding variational formulation.

According to the general theory, systems of the form (3.1) admitting a variational formulation arise from the Lagrangian density [4]

$$L(u_\Gamma) = (u_\Omega - \hat{u}_\Omega)\int_0^1 f_\Omega(\tilde{u}_\Gamma, \tilde{u}_{\Gamma,\alpha}, \tilde{u}_{\Gamma,\alpha\beta})\,\mathrm{d}\xi \tag{4.1}$$

where $\hat{u}_\Omega$ is a fixed function and $\tilde{u}_\Omega = \hat{u}_\Omega + \xi\,(u_\Omega - \hat{u}_\Omega)$; henceforth the explicit dependence on $\mathbf{X}$, t is understood. In the present case, without any loss of generality, we put $\hat{u}_i = \hat{x}_i(\mathbf{X})$ $(i = 1,2,3)$, $\hat{u}_4 = 0$; then the Lagrangian density (4.1) may be written in the form

$$L(\mathbf{x}, \phi) = \int_0^1 [(x_i - \hat{x}_i)\chi_i(\tilde{\mathbf{x}}, \tilde{\phi}) + \phi\Phi(\tilde{\mathbf{x}}, \tilde{\phi})]\,\mathrm{d}\xi$$

whence, because of (3.6),

$$L(\mathbf{x}, \phi) = \frac{1}{2}\rho_0(x_i - \hat{x}_i)\ddot{x}_i - \rho_0 b_i(x_i - \hat{x}_i) - \mu_0\phi + \int_0^1 [\phi\mathscr{D}_{M,M}(\tilde{\mathbf{x}}, \tilde{\phi}) - (x_i - \hat{x}_i)S_{iM,M}(\tilde{\mathbf{x}}, \tilde{\phi})]\,\mathrm{d}\xi. \tag{4.2}$$

It is advantageous to express the quantities $\mathbf{S}$ and $\mathscr{D}$ in terms of the scalar function Σ; indeed, in view of (3.11)–(3.13), upon choosing the gauge $\sigma = 0$, $\lambda = 0$ we have

$$S_{iM} = \frac{\partial\Sigma}{\partial x_{i,M}} + \varepsilon_{MQR}\psi_{ip,R}x_{p,Q},$$

$$\mathscr{D}_M = -\frac{\partial\Sigma}{\partial\phi_{,M}} + \varepsilon_{MQR}(\Theta_{i,R}x_{i,Q} + \Xi_{,R}\phi_{,Q}). \tag{4.3}$$

Hence it follows that

$$S_{iM,M} = \left(\frac{\partial\Sigma}{\partial x_{i,M}}\right)_{,M}, \qquad \mathscr{D}_{M,M} = -\left(\frac{\partial\Sigma}{\partial\phi_{,M}}\right)_{,M}. \tag{4.4}$$

Observe that, as always, the addition of material divergences and of material time derivatives to a Lagrangian density leaves the Euler–Lagrange equations unchanged. Accordingly, substitution of (4.4) into (4.2) allows us to write

$$L(\mathbf{x}, \phi) = \frac{1}{2}\rho_0\dot{x}_i\dot{x}_i + \rho_0 b_i x_i + \mu_0\phi + \int_0^1 \left[(x_i - \hat{x}_i)\left(\frac{\partial\Sigma}{\partial x_{i,M}}\right)_{,M}(\tilde{\mathbf{x}}, \tilde{\phi}) + \phi\left(\frac{\partial\Sigma}{\partial\phi_{,M}}\right)_{,M}(\tilde{\mathbf{x}}, \tilde{\phi})\right]\mathrm{d}\xi.$$

Finally, because

$$(x_i - \hat{x}_i)_{,M}\frac{\partial\Sigma}{\partial x_{i,M}} + \phi_{,M}\frac{\partial\Sigma}{\partial\phi_{,M}} = \frac{\partial\Sigma}{\partial\xi},$$

on disregarding an inessential material divergence, we arrive at

$$L(\mathbf{x}, \phi) = \frac{1}{2}\rho_0\dot{\mathbf{x}}\cdot\dot{\mathbf{x}} - \Sigma(\mathrm{Grad}\,\mathbf{x}, \mathrm{Grad}\,\phi) + \rho_0\mathbf{b}\cdot\mathbf{x} + \mu_0\phi. \tag{4.5}$$

It is of interest to note that, according to the Lagrangian density (4.5), the function Σ plays the role of (total) potential energy. This feature is hardly surprising although usually, in continuum mechanics, the prescription $L = T - V$ only works for conservative particle-like systems. Indeed, the structure of a particle-like system occurs just because the use of

Lagrangian coordinates preserves the similarity with a system of discrete particles. This assertion is substantiated by the variational formulation for the Lagrangian description of fluid dynamics [4] especially when compared with that for the Eulerian description [10]. In this regard we mention that the conceptual relevance of the Lagrangian description, vs the Eulerian one, in conjunction with Hamilton's principle is investigated in [11].

5. CONCLUDING REMARKS

The wide attention devoted to the thermodynamics of polarizable media motivates a brief comparison with the constitutive equations (4.3). At least in the case of the quasi-static approximation for elastic dielectrics, the restrictions placed by the second law of thermodynamics compel $\mathbf{S}$ and $\mathscr{D}$ to be given by a potential function, ε say, such that [1, 12]

$$S_{iM} = \rho_0 \frac{\partial \varepsilon}{\partial x_{i,M}}, \qquad \mathscr{D}_M = -\rho_0 \frac{\partial \varepsilon}{\partial \phi_{,M}}. \tag{5.1}$$

Now, it is apparent that the equations (4.3) are less restrictive than (5.1) in that they allow $\mathbf{S}$ and $\mathscr{D}$ to be affected by the additive terms

$$S^*_{iM} = \varepsilon_{MQR}\psi_{ip,R}x_{p,Q}, \qquad \mathscr{D}^*_M = \varepsilon_{MQR}(\Theta_{i,R}x_{i,Q} + \Xi_{,R}\phi_{,Q}).$$

To our knowledge these subtle terms have not yet been exhibited in any variational or thermodynamic approach. This should come as no surprise as long as $\mathbf{S}^*$ and $\mathscr{D}^*$ do not enter the balance equations, because

$$S^*_{iM,M} \equiv 0, \qquad \mathscr{D}^*_{M,M} \equiv 0,$$

nor do they enter the expression (4.5) for the Lagrangian density. Accordingly, they may represent, for example, a uniform stress due to external loads on the boundary and a uniform electric displacement due to a superimposed electric field.

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Résumé :

On analyse les équations régissant le comportement d'un diélectrique élastique dans l'approximation quasi statique en liaison avec le probleme inverse du calcul des variations. D'abord, par compatibilité avec les conditions d'existence de potentiel, on établit les équations fondamentales les plus générales qui permettent au modèle d'accepter une formulation variationnelle. Ensuite on détermine la forme explicite de la densité du Lagrangien. Conformément à l'adoption des coordonnées de Lagrange, la densité du Lagrangien attribue au diélectrique la même structure qu'un système de particules.

Zusammenfassung:

Die das Verhalten eines elastischen Dielektrikums beschreibenden Gleichungen werden in einer quasi-statischen Annaeherung in Verbindung mit dem umgekehrten Problem der Variationsrechnung untersucht. Zuerst werden durch die Vertraeglichkeit mit den Potentialbedingungen die Bedingungsgleichungen in allgemeinster Form hergeleitet, wodurch eine Variationsformulierung in das Modell eingefuehrt werden kann. Dann wird die explizite Form der Lagrangschen Dichte bestimmt. Folgerichtig mit der Einfuehrung Lagrangscher Koordinaten schreibt die Lagrangsche Dichte dem Dielektrikum die Struktur eines Partikelsystems zu.